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# The Three Families from $SU(4)_A \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_X$ SM-like Chiral Models.

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## Abstract

We give a detailed description of the model construction procedures about our new approach to the family structure of the standard model. SM-like chiral fermion spectra, largely "derivable" from the gauge anomaly constraints, are formulated in a $SU(N) \otimes SU(3) \otimes SU(2) \otimes U(1)$ symmetry framework as an extension of the SM symmetry. The $N = 4$ case gives naturally three families as a result, with $U(1)_Y$ nontrivially embedded into the $SU(4)_A \otimes U(1)_X$. Such a spectrum has extra vector-like quarks and leptons. We illustrate how an acceptable symmetry breaking pattern can be obtained through a relatively simple scalar sector which gives naturally hierarchical quark mass matrices. Compatibility with various FCNC constraints and some interesting aspects of the possible phenomenological features are discussed, from a non-model specific perspective. The question of incorporating supersymmetry without putting in the Higgses as extra supermultiplet is also addressed.

## I. INTRODUCTION

The matter content of the phenomenologically very successful standard model(SM) consists of three sets (families) of 15 chiral fermion states of identical quantum numbers. In addition, a scalar doublet Higgs is needed to break the electroweak(EW) symmetry and give the fermions masses. The existence of the scalar leads to the hierarchy problem, which is widely believed to be addressed by supersymmetry(SUSY). SUSY then doubles the particle spectrum, giving, in particular, scalar partners to fermions. Moreover, one extra Higgs doublet is needed; and the Higgses get fermionic partners. This makes up the minimal supersymmetric SM (MSSM). The representation structure of a single family of chiral fermions is very strongly constrained by the requirement of cancellation of all gauge anomalies, making it easily "derivable" once some simple assumptions are taken [1,2]. However, the existence of *three* families, with the great hierarchy of masses among the fermions after EW-symmetry breaking, remains a mystery.

In a short letter earlier [3], we introduced a new approach to this mysterious family problem. The approach is based on an attempt to mimic the highly contrained group representation structure of the one-family SM fermion spectrum while extending the gauge symmetry. The motivation is to look for a similarly structured list of chiral representations that, upon breaking of the extra symmetry, gives the three SM families naturally as the low-energy chiral spectrum. A general $SU(N) \otimes SU(3) \otimes SU(2) \otimes U(1)$ gauge symmetry was considered, with a successful $SU(4)_A \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_X$ model presented. A comparison with other approaches to the family problem was also given. A more ambitious idea also discussed is to incorporate SUSY without introducing extra chiral supermultiplets.

Here in this paper, we would like first to report on the details of our model construction procedures (section-II). The fact that the two essential desirable features, namely a chiral spectrum similarly structured as the one-family SM and free from all anomalies and a natural embedding of the three SM families, can be incorporated into one model is very nontrivial. However, our approach does have some flexibility that allows modifications on a

basic framework. We will present four explicit $SU(4)_A \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_X$ models obtained from slightly different modifications. Some comments on their relative merits can be found in the subsequent sections.

The models hold promises for an interesting phenomenology, provided that at least some of the extra symmetries are broken at a relative low energy scale. We will also try here to take a first look into the possible phenomenological features. However, we will try to confine our discussions mainly to the more general features of the approach, instead of the specifically model-dependent ones. Our aim is to illustrate how this new approach to the family structure can lead to interesting and potentially very successful models, and to provide useful guidelines for future detailed model constructions.

While our approach is essentially different from simply extending the SM gauge symmetry with a horizontal symmetry [4–6], part of the extra gauge symmetry obtained do behave like horizontal symmetries and can be analyzed from a similar phenomenological perspective.

In section-III, we will discuss how an acceptable symmetry breaking pattern can be obtained through a relatively simple scalar sector. We will see that this can be naturally arranged in such a way that gives SM quark mass matrices with a desirable hierarchical pattern, while evading the flavor changing neutral current(FCNC) constraints. In section-IV, we will look at the gauge sector, with its FCNC constraints and some features of the renormalization-group(RG)-runnings of the gauge couplings; the question of incorporating SUSY; and some interesting aspects of the leptonic sector. We will then make some concluding remarks in the last section.

## II. SM-LIKE CHIRAL SPECTRUM

### A. One family standard model

We start with the following perspective on the elegance of the SM representation structure for a single family.

- One can start by introducing the simplest multiplet that transforms nontrivially under each of the component group factors, namely a $(\mathbf{3},\mathbf{2},\mathbf{1})$ of $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$.(Fixing the hypercharge as 1 just corresponds to a arbitrary choice of normalization, here differs from the standard normalization by a factor of 6. We are going to stick with this particular normalization throughout the paper.)

- To cancel the $SU(3)$ anomaly, two $\bar{\mathbf{3}}$'s are needed. Keeping with the chiral structure, we have to use a $(\bar{\mathbf{3}},\mathbf{1},\mathbf{x})$ and a $(\bar{\mathbf{3}},\mathbf{1},\mathbf{y})$, with the hypercharges yet to be specified.

- Next, cancellation of the global-$SU(2)$ anomaly dictates the inclusion of an extra doublet, $(\mathbf{1},\mathbf{2},\mathbf{a})$.

- We still have to cancel all the $U(1)$-anomalies. We have $a=-3$ from the $[SU(2)]^2U(1)$ anomaly constraint. $x$ and $y$ then have to satisfy the three remaining contraints and no solution can be found. If we then allow one singlet state, $(\mathbf{1},\mathbf{1},\mathbf{k})$, we have three equations for three unknowns.

$$x+y=-2 \tag{1}$$

$$3x+3y+k=0 \tag{2}$$

$$3x^3+3y^3+k^3=45 \tag{3}$$

  This gives a unique solution, the SM hypercharge assignment $(x,y=2,-4;k=6)$. Notice that the solution *a priori* may not give a set of rational numbers. The triviality of solution here is a bit deceiving.

- A interesting fact is: if we allow two singlets, there is a one parameter set of solution with $x,y=2-n,-4+n$ and hypercharges for the singlets given by $6-n$ and $n$ ($n$ being any integer).

### B. $SU(N) \otimes SU(3) \otimes SU(2) \otimes U(1)$ extensions

We want to try to mimic the above feature in a extended symmetry that can incorporate the three families naturally. Consider adding one more component group factor, a $SU(N) \otimes SU(3) \otimes SU(2) \otimes U(1)$ guage group. $N = 4$ is obviously very suggestive. Nevertheless, let us first not fix $N$ and neglect for the moment the $U(1)$-charges. We will also suspend about the SM-embedding, with the hope to learn more about the general features to the kind of representation structures.

- We start with a $(\mathbf{N}, \mathbf{3}, \mathbf{2})$ and first take the list: $(\mathbf{N}, \mathbf{3}, \mathbf{2}), (\bar{\mathbf{N}}, \bar{\mathbf{3}}, \mathbf{1}), (\bar{\mathbf{N}}, \mathbf{1}, \mathbf{2}), (\bar{\mathbf{N}}, \mathbf{1}, \mathbf{1})$. This gives an equal number of $\mathbf{N}$'s and $\bar{\mathbf{N}}$'s and hence free us from the $SU(N)$-anomaly.

- The $SU(3)$-anomaly then suggests that there is a deficiency in $N$ $\bar{\mathbf{3}}$'s. If we take only one $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{2})$, as suggested most naturally by the pattern, we would have to take $N-2$ $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})$'s. In general, if we can allow $m$ $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{2})$'s with $N - 2m$ $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})$'s, for any $2m \leq N$.

- With a fixed choice of the numbers of $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{2})$'s and $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})$'s, we have just to count the number of $SU(2)$ doublets and add one $(\mathbf{1}, \mathbf{1}, \mathbf{2})$ if the count is odd. That caters for the global $SU(2)$-anomaly. From this perspective, one can always add in any extra even number of $(\mathbf{1}, \mathbf{1}, \mathbf{2})$'s without upsetting the anomaly constraints. The more appealing idea is of course to stay with the a minimal content.

- To complete the list of representation structures, $U(1)$-charges have to be assigned to each representations in such a way that all the gauge anomalies involving the $U(1)$ be satisfied. However, we would also like to add in pure singlets, $(\mathbf{1}, \mathbf{1}, \mathbf{1})$'s. Sticking to the SM pattern suggests taking just one, while a model with any number of singlet states is admissible.

- For example, if we take $N = 4$, we would arrive at the natural list:

$$(\mathbf{4}, \mathbf{3}, \mathbf{2}, \mathbf{1}), (\bar{\mathbf{4}}, \bar{\mathbf{3}}, \mathbf{1}, \mathbf{x}), (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}, \mathbf{y}), (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1}, \mathbf{z}),$$

$$(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{2}, \mathbf{a}), (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1}, \mathbf{b}), (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1}, \mathbf{c}),$$
$$(\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{k}), (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{s}).$$

The $U(1)$-related anomaly constraints give the conditions:

$$(3x + 2y + z) + 6 = 0; \tag{4}$$

$$4x + (2a + b + c) + 8 = 0; \tag{5}$$

$$4y + 3a + k + 12 = 0; \tag{6}$$

$$(12x + 8y + 4z) + (6a + 3b + 3c) + 2k + s + 24 = 0; \tag{7}$$

$$(12x^3 + 8y^3 + 4z^3) + (6a^3 + 3b^3 + 3c^3) + 2k^3 + s^3 + 24 = 0. \tag{8}$$

This is a list of five equations with eight unknowns. It looks like there are many solutions; but again getting a set of, reasonably small, rational numbers as a solution may be nontrivial. Simple solution does exist, for example ($x = -2, y = -1, z = 2, a = -2, b = 2, c = 2, w = -2, k = 4$) satisfies the equations. However, unlike the SM case, there is no reason to think that the solution is in any sense unique. Obviously, situations for other $N$ values are similar.

The analysis suggests an interesting pattern of SM-like chiral spectrum similarly constrained by, or "derivable" from the anomaly cancellation conditions. Of course we still have to build a connection to SM phenomenology.

### C. Embedding the three-family SM

Here, we come back to the domain of real world particle physics. We know that if any $SU(N) \otimes SU(3) \otimes SU(2) \otimes U(1)$ model has to describe nature, it has to incorporate the three-family SM as a low-energy effective field theory.

- A trivial embedding of the SM symmetry gives the $SU(N)$ component as a pure horizontal symmetry. The latter is a familiar idea [4–6], however, a horizontal symmetry model with the kind of SM-like chiral spectrum from our approach has not been proposed before. We note that $N = 3$ does give a natural three family pattern, as shown in Table 1. However, the $[SU(3)_H]^2U(1)_Y$-anomaly cannot be canceled, without modifying the spectrum.

- The next alternative is to have only the $U(1)_Y$, the (EW-)hypercharge, embedded nontrivially. We denote the symmetry by $SU(N)_A \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_X$. Note that the $SU(N)_A$, together with the $U(1)_X$, contains the hypercharge and is therefore *not* a purely horizontal symmetry. Following the argument of section-II.B, we see that the possible net numbers of quark doublets and singlets are given by $N - m$ and $2N - 2m$ respectively. Once the three families of quarks are successfully embedded, the correct number of chiral leptons would be easy to obtain, though one may have to relax the number of $(\mathbf{1}, \mathbf{1}, \mathbf{2})$ and $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ states. Hence, we require

$$N - m = 3; \quad (2N - 2m = 6); \quad 2m \leq N.$$

  Acceptable solutions are given by

$$(N, m) = (4, 1), \quad (5, 2), \quad \textit{and} \quad (6, 3).$$

  There is also an $(N, m) = (3, 0)$ trivial embedding solution mentioned above. The three solutions otherwise cannot allow a trivial embedding. Relaxing the $2m \leq N$ criteria, and allowing $(\mathbf{1}, \mathbf{3}, \mathbf{1})$'s instead of $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})$'s, then solution is possible for all $N$. Obviously, the $N = 4$ case is singled out as the most interesting and gives three quark families most naturally. For instance, $(N, m) = (3, 1)$ gives two quark families, while $(N, m) = (5, 1)$ gives four. $N = 4$ is also the only one that gives naturally three leptonic doublets. See Table 1. for an illustration.

- One can also consider nontrivial embedding of the $SU(3)_C$ and $SU(2)_L$ component factors. At least for $N \leq 6$, a SM-like chiral spectrum fails to give three families.

- To complete the SM embedding, we need to fix the explicit hypercharge embedding. This is done for the $N = 4$ case in the next section.

### D. Illustrative $SU(4)_A \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_X$ models

Now, we stick to the most interesting case of $N = 4$ and analyse all possible embedding of $U(1)_Y$ into $SU(4)_A \otimes U(1)_X$.

- There are three independent $U(1)$ subgroups in a $SU(4)$. We choose to consider $SU(2)_H \otimes SU(2)_K \otimes U(1)_Z \subset SU(4)_A$ with the two extra $U(1)$ interpreted as the diagonal generators of the two $SU(2)$'s. The $U(1)_Y$ can be any linear combination of the three and the $U(1)_X$. To fix a convention, we take $\mathbf{4} \longrightarrow (\mathbf{2}, \mathbf{1})_{-\mathbf{1}} + (\mathbf{1}, \mathbf{2})_{\mathbf{1}}$.

- In Table 2, we listed the the $U(1)_Y$ states, following the list of representations and notations of section-II.B, with

$$U(1)_Y = \alpha U(1)_X + \beta U(1)_Z + \gamma U(1)_H + \delta U(1)_K. \tag{9}$$

  The $U(1)_H$ and $U(1)_K$ here are the $U(1)$-subgroups of the correspondent $SU(2)$'s; in particular,

$$U(1)_{H,K} = 2T_{3(H,K)} \; . \tag{10}$$

  To get a three-family structure for the quarks, we can set, without lose of generosity,

$$\gamma = 0, \tag{11}$$

$$\delta = -2\beta, \tag{12}$$

$$a\alpha = -\alpha - 3\beta, \tag{13}$$

  and

$$b\alpha = c\alpha = x\alpha - 3\beta. \tag{14}$$

Similarly, we obtain for the leptonic sector

$$k\alpha = -y\alpha + 3\beta \tag{15}$$

and

$$s\alpha = -z\alpha + 3\beta. \tag{16}$$

The three-family structure is shown in the last column of the table. Note that the structure is compatible with a

$$\begin{aligned} SU(4)_A \otimes U(1)_X &\longrightarrow SU(3)_H \otimes U(1)_{Z'} \otimes U(1)_X \\ &\longrightarrow SU(3)_H \otimes U(1)_Y \end{aligned}$$

symmetry embedding as used in our earlier presentation [3]. Both the $SU(3)_H$ or the $SU(2)_H$ may then serve as a real horizontal symmetry for the SM under this framework.

- To get the correct hypercharges, we have, for the three quark doublets

$$\alpha - \beta = 1, \tag{17}$$

and for the quark singlets

$$x\alpha + \beta = 2 \quad (or \quad -4), \tag{18}$$

and

$$x\alpha - 3\beta = -4 \quad (or \;\; 2). \tag{19}$$

We denote hereafter the two different quark-singlets embeddings, given through the two hypercharge identifications as shown in the equations, as schemes $I$ and $II$ respectively. For the leptons, we require

$$y\alpha + \beta = -3 \tag{20}$$

and

$$z\alpha + \beta = 6. \tag{21}$$

Each of the two schemes gives a unique solution:

$$I : \alpha = 5/2, \beta = 3/2; x = 1/5, y = -9/5, z = 9/5;$$
$$a = -14/5, b = c = -8/5; k = 18/5, s = 0;$$

$$II : \alpha = -1/2, \beta = -3/2; x = 5, y = 3, z = -15;$$
$$a = -10, b = c = -4; k = 6, s = 24.$$

So far in the analysis of the possible SM embeddings, we have not imposed the $U(1)_X$-related gauge anomaly constraints listed in the equations of section-II.B. For any of the two embedding solutions to give a consistent model, we need to check the anomalies. It looks like we need a miracle to have the conditions all just satisfied. It does *not* work. However, we are close. For both solutions, three equations, apart from the first and last on the list, are satisfied. This suggests a slight modification of the chiral spectrum may get around the problem and give interesting models.

- *(Model IA)* We first point out that there is a sextet representation in $SU(4)$ that is anomaly free. Taking the scheme $I$ solution, the simplest modification then is to introduce a $(\mathbf{6}, \mathbf{1}, \mathbf{1}, -\mathbf{12/5})$. Hereafter we change the normalization for the $U(1)_X$-charges by a factor of 5 (only for the scheme), giving all of them integral values, for convenience ( *i. e.* the sextet is then a $(\mathbf{6}, \mathbf{1}, \mathbf{1}, -\mathbf{12})$, for instance). The representation then fixes the $[SU(4)_A]^2U(1)_X$-anomaly and leads to three extra leptonic singlets of hypercharges $-\mathbf{3}$ and three of $-\mathbf{9}$ (following the same normalization as in section-II.A). To restore a three family SM chiral spectrum above the EW-scale, the vector-like

partners of these singlets can be introduced as $(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{6})$'s and $(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{18})$'s. This happens to just cancel the other $U(1)_X$-anomalies and gives a consistent model. The details of the representation content of the model are shown in Table 3. We also show all the $U(1)_X$-related anomalies as an independent checking of the result. One interesting state here is the pure SM-singlet ($N$) in $(\bar{\mathbf{4}},\mathbf{1},\mathbf{1},\mathbf{9})$. If we consider a SUSY-version of the model and promote the fermionic multiplets to chiral supermultiplets, there is naturally a Yukawa coupling

$$(\mathbf{4},\mathbf{3},\mathbf{2},\mathbf{5})\ \ (\mathbf{1},\bar{\mathbf{3}},\mathbf{2},-\mathbf{14})\ \ (\bar{\mathbf{4}},\mathbf{1},\mathbf{1},\mathbf{9})$$

with VEV for the scalar partner of the state that gives the symmetry breaking $SU(4)_A \otimes U(1)_X \longrightarrow SU(3)_H \otimes U(1)_Y$ as well as mass to the vector-like quark doublet($Q'$). We will return to this in the other sections below.

- *(Model IIA)* Same as in the scheme *I* solution, we take the scheme *II* solution and add a $(\mathbf{6},\mathbf{1},\mathbf{1},\mathbf{24})$ and three $(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{12})$'s. A successful model is resulted. The desirable Yukawa term, analog to the one listed above, is not allowed though. Nevertheless, there is one desirable feature for the SUSY-version which does not exist for *Model I*: we have the vector-like pair of leptonic doublets identifiable with the Higgs(ino) supermultiplets of MSSM.

- *(Model IIB)* For the scheme *II* solution. A $z$-value of 9 is again needed for the above mentioned Yukawa term,

$$(\mathbf{4},\mathbf{3},\mathbf{2},\mathbf{1})\ \ (\mathbf{1},\bar{\mathbf{3}},\mathbf{2},-\mathbf{10})\ \ (\bar{\mathbf{4}},\mathbf{1},\mathbf{1},\mathbf{9})$$

in this case, to be admissible. The content of singlets has then to be modified to cancel the gauge anomalies. The resulted model is the one we presented earlier [3].

We summarize the contents of the three models in Table 4, together with an extra, minimal model, to be introduced below. Though we noted above the possible inclusion of

the symmetry breaking scalars through supersymmetrizing, the discussion in the section should however be taken as addressing mainly the fermionic sector. We will look at the possible symmetry breaking patterns and the related scalar sectors in section-III.

### E. Minimal models and variations on the theme

We can look at the model construction exercises this way: one can start with a $SU(N)_A \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_X$ SM-like chiral spectrum as shown in Table 1 for $N = 3, 4, 5$ and 6, with the modification that two, or four, extra $(\mathbf{1}, \mathbf{1}, \mathbf{2})$'s ($U(1)_X$-charge suppressed), have to be added for the cases $N = 5$ and 6, and some singlets for all cases. The $U(1)_X$-charges are then determined by the possible $U(1)_Y$ hypercharges embeddings as discussed above. Then only the $[SU(N)_A]^2 U(1)_X$, the $U(1)_X$-grav. and the $[U(1)_X]^3$ anomalies may be uncanceled. One can add in a pure, anomaly-free, $SU(N)_A$-representation, irreducible or reducible, and adjust its $U(1)_X$-charge(s) to cancel the $[SU(N)_A]^2 U(1)_X$-anomaly. Finally, add in singlets with the proper $U(1)_X$-charges to restore the three-family SM chiral spectrum at the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ level. All the other anomalies are canceled. This is actually a consequence of the embedding and the fact that the SM is anomaly-free.

As an alternative to adding extra $SU(N)_A$-representation(s), one can also give up the embedding of the leptonic singlets in the $(\bar{\mathbf{N}}, \mathbf{1}, \mathbf{1})$ and simply adjust the $U(1)_X$-charge of the lattter to fix the $[SU(N)]^2 U(1)$-anomaly and proceed as above, putting in the leptonic singlets as singlets. This actually yields models with the minimal total number of states. The minimal model for $N = 4$ scheme $I$ embedding is listed as *Model Im* in Table 4, for an illustration. $N = 5, 6$ and 3 cases are given in Table 5, Table 6 and Table 7 respectively. The minimal models, apart from the $N = 3$ case, may however give SM-singlets states with unnaturally large hypercharges, as shown in the tables [7]. They should be considered more as backgrounds for making modifications, as discussed here, to obtain potentially interesting models.

Note that for each $N$, there are always two alternate hypercharge embeddings for the

quark singlets, labeled by scheme $I$ and $II$, as for $N = 4$ shown explicitly in section-II.D. For $N = 6$ (Table 6), we have assumed that the $\mathbf{6}$ and the $\bar{\mathbf{6}}$'s each splits into two groups each with three states of the same hypercharges. This may be considered, for instance, as taking $SU(6) \longrightarrow SU(3) \otimes SU(3) \otimes U(1)_Z$ with $U(1)_Y$ given by a linear combination of $U(1)_Z$ and $U(1)_X$. Hence each of scheme $I$ and $II$ gives two possible embeddings for both the quark doublets and leptonic doublets, depending on which three states are identified as the SM chiral states. The quark doublet embedding ambiguity may be absorbed by a sign convention. Then, the two leptonic doublet embedding identifications give two different sub-schemes for each of scheme $I$ and $II$. $N = 5$ has a similar situation (Table 5). Though one group from the splitting of a $\mathbf{5}$ or a $\bar{\mathbf{5}}$ has two instead of three states, we allow the alternative of identifying the two states as the SM leptonic doublets. This gives alternate models with four extra $(\mathbf{1}, \mathbf{1}, \mathbf{2})$, instead of two for all the other $N = 5$ and 6 schemes.

If one is willing to go into more complicated variations, one can still temper with the $SU(2)_L$-doublet sector, by giving up the embedding of the leptonic doublets into the $(\bar{\mathbf{N}}, \mathbf{1}, \mathbf{2})$ for instance, and get modified models without too much difficulty. Nevertheless, more modifications usually introduce more states and make the pattern deviate more from the starting theme of a SM-like chiral spectrum.

Finally, we comment on the $N = 3$ case. The minimal models are shown in Table 7. The two models, or their modified versions, have a $SU(3)_H (\equiv SU(3)_A)$ horizontal symmetry with the $U(1)_X$ identified directly as the hypercharge $U(1)_Y$. While $SU(3)_H$ was among the first group to be considered as a horizontal symmetry for the three-family SM, the chiral fermion content, with right-handed neutrinos, was considered to be vector-like in the $SU(3)_H$ [6]. Our models here start with a SM-like chiral spectrum and have a very different basic structure. Whether this kind of $SU(3)_H$ models can have a successful phenomenology we leave for further investigation.

In the rest of the paper, we will put our concentration back on the $N = 4$ case, which we consider most natural in the framework and most illustrative of the general features of our approach.

Before closing the section, we note that a $SU(5) \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ model has recently been proposed in a different perspective [8]. Our approach here is partially inspired by the model. Also after completing the work presented in this section, the author was informed about a $SU(2)_L \otimes SU(2)_R \otimes SU(3)_C \otimes SU(4)_G$ model [9] which also has a partial embedding of $U(1)_Y$ into the $SU(4)_G$. The model construction is otherwise very different from our approach. We emphasize that a $SU(4)_A \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_X$ model from our approach has a chiral fermion spectrum that is mostly derived from anomaly constraints and gives naturally three SM families as a result. This is its unique interesting feature.

## III. THE QUARK SECTOR AND THE SYMMETRY BREAKING SCALARS

We have presented in the previous section explicitly four $SU(4)_A \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_X$ models with SM-like chiral fermion spectra (see also Table 4), from two different basic schemes of embedding the three-family SM (scheme *I* and *II* solutions). In this section we address the important question of how the extra symmetries can be broken in a way that gives an experimentally viable low-energy phenomenology. This is hooked-up with fermion mass generation through EW-symmetry breaking. We hence discuss here also the prospect of getting some realistic quark mass matrices. Also of interest is the possible phenomenology of the extra heavy quarks.

### A. Symmetry breaking and quark masses

What we need, first of all, are scalars with VEVs that break $SU(4)_A \otimes U(1)_X$ to $U(1)_Y$, with or without an intermediate horizontal symmetry (*i. e.* through a $G_H \otimes U(1)_Y$). Such scalars must be pure SM-singlet states (with hypercharge zero). Then we need the EW-Higgs doublet(s). The latter, together with the singlet scalars, has to generate the quark masses with a nature hierarchical structure. Particularly, the SM quark mass matrices have

to be rank-one to first order, and the extra quarks ($Q'$ or $Q''$) that are vector-like under SM-group should be heavy.

So far we have not restricted our discussion to any specific model. Attentive readers would have realized that the quark sector structure is robust within each embedding scheme; that is still quite true even when we compare the cases with $N = 5$ and 6 to the $N = 4$ one. Also recall that the two basic embedding schemes differ only in the way the $\bar{u}$ and $\bar{d}$ states are embedded. The major difference is that the scheme *I* models (e. g. *Model IA* and *Im*) have an extra vector-like quark doublet ($Q'$) with electric charges $(5/3, 2/3)$, while the scheme *II* models (e. g. *Model IIA* and *IIB*) have one ($Q''$) with electric charges $(-1/3, -4/3)$. The former contain an extra up-type quark and the latter a down-type one, both with the "wrong" isospin. Then it is no surprise that the family structure of the scheme *I* up-sector looks like that of the scheme *II* down-sector, and vice versa. This specific feature allows us to talk about the symmetry breaking issue and the quark mass matrices from both schemes together. We have a three-quark sector and a four-quark sector; the former is the down-sector and the latter the up-sector for scheme *I*, while their identities switch for scheme *II*. The fourth quark in the four(-quark)-sector, the guy with the "wrong" isospin, is part of the vector-like doublet. Its partner is an extra quark, of electric charge 5/3 or $-4/3$, that does not mix with any of the others.

We again consider $SU(2)_H \otimes SU(2)_K \otimes U(1)_Z \subset SU(4)_A$ and use the quantum numbers in the subgroup embedding as a convenient label for the states. For instance, the $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1}, \mathbf{9})$ contains the four states denoted by $(\mathbf{2}_\pm, \mathbf{1})_{\mathbf{1},\mathbf{9}}$ and $(\mathbf{1}, \mathbf{2}_\pm)_{-\mathbf{1},\mathbf{9}}$ where the $(\mathbf{1}, \mathbf{2}_+)_{-\mathbf{1},\mathbf{9}}$ state has $T_{3(K)} = +1/2$ and $U(1)_Z = -1$. ( $U(1)_X = 9$; here in the notation we suppress the $SU(3)_C$ and $SU(2)_L$ quantum numbers.) From the results of section-II.D, the hypercharges are then given by

$$U(1)_Y = \pm \left( \frac{1}{2} U(1)_X + \frac{3}{2} \left[ U(1)_Z - 4T_{3(K)} \right] \right) \tag{22}$$

with the positive and negative signs corresponding to the scheme *I* (with above modified $U(1)_X$ normalization) and *II* solutions respectively. Our example state $(\mathbf{1}, \mathbf{2}_+)_{-\mathbf{1},\mathbf{9}}$ is then a

state of hypercharge zero. Also, the Higgs doublets that are capable of giving masses to the SM-quarks satisfy the equation

$$\frac{1}{2}U(1)_X + \frac{3}{2}\left[U(1)_Z - 4T_{3(K)}\right] = \pm 3 \tag{23}$$

with the positive and negative signs correspond to the ones coupling to the four- and three(-quark)-sectors respectively, independent of which schemes or models we are talking about. We can now look at the particular Higgs-VEVs needed to generate mass for each entries of the quark mass matrices of both sectors. They are given by

$$M^{(3)} \sim \begin{pmatrix} \begin{matrix}\langle(\mathbf{3_0},\mathbf{1})_{\mathbf{0},\mathbf{-6}}\rangle \\ or \\ \langle(\mathbf{1},\mathbf{1})_{\mathbf{0},\mathbf{-6}}\rangle\end{matrix} & \langle(\mathbf{3}_+,\mathbf{1})_{\mathbf{0},\mathbf{-6}}\rangle & \langle(\mathbf{2}_+,\mathbf{2}_-)_{\mathbf{2},\mathbf{-6}}\rangle \\ \\ \langle(\mathbf{3}_-,\mathbf{1})_{\mathbf{0},\mathbf{-6}}\rangle & \begin{matrix}\langle(\mathbf{3_0},\mathbf{1})_{\mathbf{0},\mathbf{-6}}\rangle \\ or \\ \langle(\mathbf{1},\mathbf{1})_{\mathbf{0},\mathbf{-6}}\rangle\end{matrix} & \langle(\mathbf{2}_-,\mathbf{2}_-)_{\mathbf{2},\mathbf{-6}}\rangle \\ \\ \langle(\mathbf{2}_-,\mathbf{2}_+)_{\mathbf{-2},\mathbf{-6}}\rangle & \langle(\mathbf{2}_+,\mathbf{2}_+)_{\mathbf{-2},\mathbf{-6}}\rangle & \begin{matrix}\langle(\mathbf{1},\mathbf{3_0})_{\mathbf{0},\mathbf{-6}}\rangle \\ or \\ \langle(\mathbf{1},\mathbf{1})_{\mathbf{0},\mathbf{-6}}\rangle\end{matrix} \end{pmatrix} \tag{24}$$

and

$$M^{(4)} \sim \begin{pmatrix} \langle(\mathbf{2_-},\mathbf{1})_{\mathbf{1,3}}\rangle & \langle(\mathbf{2_+},\mathbf{1})_{\mathbf{1,3}}\rangle & \langle(\mathbf{1},\mathbf{2_-})_{\mathbf{-1,3}}\rangle & \langle(\mathbf{1},\mathbf{2_+})_{\mathbf{-1,3}}\rangle \\ \\ \langle(\mathbf{2_-},\mathbf{1})_{\mathbf{1,3}}\rangle & \langle(\mathbf{2_+},\mathbf{1})_{\mathbf{1,3}}\rangle & \langle(\mathbf{1},\mathbf{2_-})_{\mathbf{-1,3}}\rangle & \langle(\mathbf{1},\mathbf{2_+})_{\mathbf{-1,3}}\rangle \\ \\ \langle(\mathbf{2_-},\mathbf{2_-})_{\mathbf{2,-6}}\rangle & \langle(\mathbf{2_+},\mathbf{2_-})_{\mathbf{2,-6}}\rangle & \langle(\mathbf{1},\mathbf{3_-})_{\mathbf{0,-6}}\rangle & \begin{matrix}\langle(\mathbf{1},\mathbf{3_0})_{\mathbf{0,-6}}\rangle \\ or \\ \langle(\mathbf{1},\mathbf{1})_{\mathbf{0,-6}}\rangle\end{matrix} \\ \\ 0 & 0 & 0 & \langle(\mathbf{1},\mathbf{2_+})_{\mathbf{-1,9}}\rangle \end{pmatrix}, \tag{25}$$

where we noted that the $\mathbf{3}_+, \mathbf{3_0}$ and $\mathbf{3}_-$ correspond to states in a $SU(2)$ triplet with $T_3 = +1, 0$ and $-1$ respectively. As remarked above, the VEV $\langle(\mathbf{1},\mathbf{2}_+)_{\mathbf{-1,9}}\rangle$ is in a pure SM-singlet state and, if admitted, results in the symmetry breaking $SU(4)_A \longrightarrow SU(3)_H \otimes U(1)_{Z'}$ and give masses to both quarks in the vector-like doublet ($Q'$ or $Q''$). The mass term corresponds to the 44−entry of the $M^{(4)}$-matrix. With only this VEV, $SU(3)_H$ then serves as an unbroken horizontal symmetry. The VEV is hence a desirable one, though not necessarily the $SU(3)_H$. All the other VEVs shown in the mass matrices are in the zero electric charge components of doublets(EW). The zero matrix-entries correspond to states of zero electric charges that would only be available from $SU(2)_L$ triplets which we assume nonexisting. Note that the entries of the first two rows of $M^{(4)}$ are identical, giving one zero-eigenvalue. One naturally very small quark mass eigenstate is therefore to be expected.

Assuming an un-broken $U(1)_Y$, the VEV in $(\mathbf{1},\mathbf{2}_+)_{\mathbf{-1,9}}$ is the only one admissible in $\phi_0 = (\bar{\mathbf{4}},\mathbf{1},\mathbf{1},\mathbf{9})$. Actually, we can switch the argument the other way round and use the natural VEV

$$\langle\phi_0\rangle = \begin{pmatrix} 0 & 0 & 0 & v \end{pmatrix} \tag{26}$$

to define the remnant $SU(3)_H$ and $U(1)_Y$ symmetry. Other representations may then have more zero hypercharge states. The interesting thing is: all the doublet VEVs in $M^{(3)}$ and

$M^{(4)}$ listed above can come from just two representations, a $(\mathbf{15},\mathbf{1},\mathbf{2},-\mathbf{6})$ and a $(\bar{\mathbf{4}},\mathbf{1},\mathbf{2},\mathbf{3})$. One easy way to obtain rank-one EW-quark mass matrices is to take just the scalar $\Phi = (\mathbf{15},\mathbf{1},\mathbf{2},-\mathbf{6})$ and give VEVs only to the $(\mathbf{1},\mathbf{3_0})_{\mathbf{0},-\mathbf{6}}$ and $(\mathbf{1},\mathbf{3_-})_{\mathbf{0},-\mathbf{6}}$ states. The VEVs actually perserves the $SU(2)_H$ as a horizontal symmetry which keeps the lighter two families massless. The abundance of neutral scalars coupling to the quarks, here from a single Yukawa term involving $\Phi$, leads to worries about FCNC constraints [10], which basically requires most if not all the other scalar in the $\Phi$ without a VEV to be heavy ($\geq 200TeV$). But there is a way that this can be done. A $(\bar{\mathbf{4}},\mathbf{1},\mathbf{1},-\mathbf{3})$ gives three zero hypercharge states, in an anti-triplet of the $SU(3)_H$. If we take two such scalar multiplets, denoted by $\phi_a$ ($a = 1$ or 2), with natural VEVs

$$\langle\phi_1\rangle = \begin{pmatrix} v_1 & 0 & 0 & 0 \end{pmatrix}, \qquad \langle\phi_2\rangle = \begin{pmatrix} v_1' & v_2 & 0 & 0 \end{pmatrix}, \tag{27}$$

a mass term for $\Phi$ of the form

$$C_{ab}\phi_{ai}\phi_b^{\dagger j}\Phi_j^k\Phi_k^{\dagger i}$$

gives masses to all components of the scalar bearing non-trivial $SU(2)_H$ quantum numbers. This is basically the mass term used previously in a $SU(3)_H$ model [11]. It can be interpreted as enforcing the coupling of $\phi_a\phi_b^\dagger$, with their nonzero VEVs only in the $SU(2)_H$ nontrivial directions, to the $\Phi\Phi^\dagger$ only through the part that transforms as a $(\mathbf{3},\mathbf{1})_\mathbf{0}$. Recall the splitting

$$\mathbf{15} \longrightarrow (\mathbf{1},\mathbf{1})_\mathbf{0} + (\mathbf{1},\mathbf{3})_\mathbf{0} + (\mathbf{3},\mathbf{1})_\mathbf{0} + (\mathbf{2},\mathbf{2})_\mathbf{2} + (\mathbf{2},\mathbf{2})_{-\mathbf{2}}.$$

It can then be seen easily that the term splits the multiplet and leaves only the $(\mathbf{1},\mathbf{3})_\mathbf{0}$ states massless, with EW-scale masses then assumed to be generated by other mechanisms, for instance radiatively. The sub-multiplet, apart from the $\Phi_+ = (\mathbf{1},\mathbf{3}_+)_{\mathbf{0},-\mathbf{6}}$ doublet which contains no neutral states, gives the EW-breaking doublets we want. The $\Phi_+$ doublet has scalar states with electric charges $(\mp 1, \mp 2)$ [12] giving Yukawa couplings $\bar{d}\Phi_+Q'$ or $\bar{u}\Phi_+Q''$. The doubly-charged scalar state in particular can be considered a novel part of the prediction from our models.

It is interesting to see that one can go on to construct almost the full quark mass matrices even without introducing further extra scalars. For instance, combining $\Phi$ and $\phi_0$, through a dimension-5 term can give $1/M_0$ suppressed effective mass term of the form

$$\langle\Phi\rangle\, v_0/M_0$$

to four more entries in $M^{(4)}$ as shown in Table 8a, leading to a second non-zero SM quark mass eigenvalue. Here $M_0$ would have to be some mass scale higher than $v_0$. To name two possibilities: it can be $M_{Pl}$ with the effective Yukawa coupling being gravitationally generated, if $v_0$ is at a very high scale [13]; or it can be the mass scale of some other vector-like fermions with the effective coupling being generated *a la* Froggatt-Nielsen [14] (see Fig. 1 for an illustration). In a non-SUSY-compatible scenario, further combining with $\phi_a (a = 1, 2)$ can give at least a second mass eigenvalue for $M^{(3)}$ (see Table 8a). For the scheme $I$ models, this corresponds to a natural mass hierarchy

$$m_t, m_b > m_c > m_s > m_d, m_u$$

leaving only the $m_t/m_b$ ratio to be fixed by their VEVs and the very small $m_d$ and $m_u$ to be generated by radiative mechanism. There is also an alternative but similar possibility of starting with $\Phi = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}, \mathbf{3})$ and the same set of singlet scalar as shown in Table 8b. For the scheme $I$ models, this corresponds to a natural mass hierarchy

$$m_t > m_b, m_c, m_s > m_d, m_u \quad .$$

We consider the above analysis a partial success in generating the quark mass hierarchy and an illustration of the promising potentials of our approach. To complete the quark mass matrices construction, radiative mass generations have to be analyzed and explicit mass scales have to be fixed. Other scalar VEVs may also be introduced. This should be done within the context of a specific model.

### B. The heavy quarks

Recall that our approach gives a characteristic extra quark doublet ($Q'$ or $Q''$) which is vector-like under the SM group. In the discussions above, we adopt a symmetry-breaking scalar $\phi_0 = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1}, \mathbf{9})$ which gives masses $M_Q$ to the quark doublet through a Yukawa term. If the mass is at a high scale, as is to be expected for the symmetry breaking (*i. e.* $v_0 \geq 200TeV$), there would hardly be any interesting accessible phenomenology. However, if the mass for the quarks were substantiallly below symmetry breaking scale, perhaps due to a small (effective) Yukawa coupling as in the cases of the SM quarks apart from the top, it could have a much more interesting experimental implication. In this subsection, we discuss some of the interesting possibilities under the assumption.

The two constituents of the doublet behave differently: one is the fourth up- or down-type quark ($q_4$), which most propably mixes with the other light quarks in $M^{(4)}$; the other is an exotic guy, an "above-the-top" or "below-the-bottom" quark ($q_*$), with electric charge $5/3$ or $-4/3$ respectively. The two scenarios correspond to the two schemes of embedding; and the identity of this extra quark doublet is what distinguishes models of the two schemes.

The heavier of the two quarks, $q_4$ or $q_*$, can decay into the lighter and a $W$-boson, if their mass splitting is large enough. However, large mass splitting is unfavorable, from both theoretical and experimental perspectives. From the group structure of the fermion spectrum, we expect a mass degeneracy between the two quark states, only to be lifted by the small mixing between $q_4$ and the light quarks. The mass splitting is also constrained by the experimental limit on its contribution to the $\rho$-parameter. In a degenerate scenario, the major decay mode for $q_*$ is likely to involve Yukawa vertices, as the quark is assumed to be much lighter than the extra gauge bosons. For instance, if $\Phi = (\mathbf{15}, \mathbf{1}, \mathbf{2}, -\mathbf{6})$ is taken to provide the EW-Higgses, a light doublet of charged Higgses is predicted and both $q_*$ and $q_4$ can decay into both an up- or down-type quark with one of the Higgses. Under the situation of small Yukawa couplings or nonexistence of such scalars, $q_*$ is likely to be relatively stable, while $q_4$ can still decay through Yukawa couplings responsible for its mass mixing.

If the quarks were not too fast-decaying, they may, upon QCD-confinement, form new mesonic and baryonic states, among themselves or with the light quarks. All these QCD-singlet states have integral charges. The states involving $q_*$ would be more interesting; for example, there could be doubly-charged mesons.

Phenomenology of SM vector-like quarks has been studied [15] particularly extensively from the perspective of spontaneous CP violation [16], and recently in the context of fixing the $R_b - R_c$ anomaly [17]. The type of analysis is particularly relevant for the quark $q_4$. CP violation has also been ascribed to horizontal interactions by other authors [18]. The extra gauge symmetries in our approach has some similarities to horizontal symmetries, while our models also have the vector-like quarks. CP nonconservation properties of models from our approach would be very interesting. We will, however, leave the issue to further investigation. We discuss here FCNC constraints and possible impact on the $R_b - R_c$ anomaly. Most of the result can be easily adopted from the cited references.

The kind of Higgs configurations for quark mass generation we discussed above has basically the Natural Flavor Conservation feature in it at the EW-scale. However, the mass mixing involving $q_4$ in $M^{(4)}$ does generate FCNC's at the $Z$-vertex, essentially because of the "wrong" isospin of $q_4$. Taking a biunitary rotation to diagonalize the first $3 \times 3$ block, we can write

$$
\begin{aligned}
\tilde{M}^{(4)} &= \begin{pmatrix} U_R^\dagger & \\ & 1 \end{pmatrix} M^{(4)} \begin{pmatrix} U_L & \\ & 1 \end{pmatrix} \\
&\sim \begin{pmatrix} m_1 & & & x_1 \\ & m_2 & & x_2 \\ & & m_3 & x_3 \\ 0 & 0 & 0 & M_Q \end{pmatrix} .
\end{aligned} \tag{28}
$$

In terms of the mass eigenstates $q^{'i}$

$$
L_Z^{FCNC} = \beta_{ij} \bar{q}_R^{'i} \gamma_\mu q_R^{'j} Z^\mu \tag{29}
$$

for $i \neq j$, where

$$\beta_{ij} = \left(\mp\frac{1}{2}\right)\frac{g_2}{cos\theta_W}(K_R)^*_{4i}(K_R)_{4j} \tag{30}$$

with $K_R^\dagger \tilde{M}^{(4)} K_L = diag\{M^{(4)}\}$ and the signs correspond to the two schemes [12]. Here the more interesting situation comes from scheme $II$ embedding where the four-sector is the down-sector. There, a stronger constraint coming from Kaon decay $K_L \longrightarrow \mu^+\mu^-$ search requires $\beta_{12} < 10^{-6}$ [19]. Now

$$(K_R)^*_{4i} = x_i/M_Q = (U_R^\dagger)_{ij} M^{(4)}_{j4}/M_Q; \tag{31}$$

and from the group symmetry structure of $M^{(4)}$ we expect then

$$x_1 \sim x_2 < 10^{-3} M_Q. \tag{32}$$

Putting $x_2 = m_s$ implies only $M_Q > 150 GeV$. Corresponding FCNC contributions from the left-handed component is much further suppressed as

$$(K_L)^*_{4i} = \frac{x_i}{M_Q}\frac{m_i}{M_Q}. \tag{33}$$

$x_3$, however, is expected to be larger. Actually, the group symmetry structure gives

$$x_3 = -M^{(3)}_{33} \tag{34}$$

which, in the case of scheme $II$ models, gives

$$x_3 \sim m_t. \tag{35}$$

The mixing serve in the right direction to fix the $R_b$-anomaly. Quantitatively, we need

$$\left(\frac{x_3}{M_Q}\right)^2 = 0.059 \pm 0.016, \tag{36}$$

giving a value of $M_Q$, or rather $m_{q_4}$ in particular, to be

$$m_{q_4} \sim 635 - 840 GeV. \tag{37}$$

In the scheme $I$ models, mixing with $q_4$ goes to the up-sector. FCNC constraints are much less severe. However, $x_2$ can be used to reduce $R_c$ only in the context of large mixing,

and $x_3$ can be used to give large mixing with the top which could then reduce $R_b$ through changing the top-loop effect [20]. The scenario flavors a much lighter $q_4$, perhaps below $m_t$ pulling the ratio between $M_Q$ and the symmetry breaking scale much smaller. That makes it very unlikely to fit in with other phenomenological aspect for a complete model from our approach and therefore less attractive.

## IV. THE GAUGE SECTOR, AND MORE

### A. The gauge sector

The gauge bosons of $SU(4)_A$ are contained in an adjoint **15**, nine among them are neutral. One among the nine, the one that transforms as $(\mathbf{1},\mathbf{1})_\mathbf{0}$ in the case of a simple $SU(4)_A \longrightarrow SU(2)_H \otimes SU(2)_K \otimes U(1)_Z$ symmetry breaking, mixes with the $U(1)_X$-boson to give an heavy state and a massless one. The latter is to be identified as the $U(1)_Y$-boson when the symmetry is broken to that of the SM. The heavy state together with the other eight, which also develop heavy masses, behave very much like gauge bosons of horizontal intereactions and have to satisfy similar FCNC constraints [21,22]. The other six states of the adjoint **15** are states of non-zero $T_{3(K)}$ and correspond to charged gauge-bosons: $(\mathbf{1},\mathbf{3}_-)_\mathbf{0}$ and $(\mathbf{2}_\pm,\mathbf{2}_-)_\mathbf{2}$ of charge $\pm 1$, and their conjugates $(\mathbf{1},\mathbf{3}_+)_\mathbf{0}$ and $(\mathbf{2}_\pm,\mathbf{2}_+)_{-}\mathbf{2}$ of charge $\mp 1$ [12]. Such gauge bosons are *not* to be expected in a horizontal symmetry framework. The charged gauge bosons can contribute to FCNC only through loop-diagrams. As they are expected to have masses at the same scale with the neutral ones, such contribution would play a secondary role.

The strongest FCNC constraint relevant is given by the process $K_L \longrightarrow \mu e$ which gives lower bound on the neutral gauge boson mass [19] as

$$M_{\mathcal{X}} \sim 220TeV \left[\frac{10^{-12}}{B(K_L \to \mu e)}\right]^{1/4} , \tag{38}$$

comparable to that on the heavy neutral scalars discussed in the section-III.A above. A potentially stronger bound coming from $K_L - K_S$ mass difference is liable to a reduction

factor of $\Delta_{M_\mathcal{X}}/M_\mathcal{X}$ where the numerator denote the lack of degeneracy among the gauge boson masses [21].

The extra charged gauge bosons give extra contributions to charge current processes with special characteristic. For instance, for the SM quarks, the couplings involve only a $\bar{u}$ and a $\bar{d}$ in the same $(\bar{\mathbf{4}}, \bar{\mathbf{3}}, \mathbf{1}, \mathbf{x})$ multiplet (see Fig.2); with the type of Higgs structure discussed above, at least one of them would be predominately a third family guy. The processes are of course suppressed by the same gauge boson mass scale $M_\mathcal{X}$.

Another interesting aspect concerning the gauge sector is the RG-runnings of the gauge couplings. While there is no obvious gauge group unification, string-type gauge coupling unification may not be ruled out. The RG-runnings consideration is likely to impose limits on the acceptable mass scales involved. One particularly interesting aspect under the assumption of gauge coupling unification is that if the $SU(4)_A$ is asymptotically free, to get the correct count of the number of families, we would have to make sure that it breaks before it confines. With a strong asymptotic freedom, this may even set a pretty high lower limit on the symmetry breaking scale, making the extra vector-like fermions such as the quark doublet ($Q^{'}$ or $Q^{''}$) less likely to be accessible to low-energy phenomenology. The RG-runnings are of course dependent on the specific details of the models including the full content of the scalar multiplets, which may include extra ones needed to give masses to some of the leptonic sector states. We are hence going to just sketch briefly some possible scenarios.

As an example, taking a supersymmetrized version of the spectrum of *Model IIB*, without extra scalar, the coefficients for the first order $\beta$-functions are given by

$$(b_4, b_3, b_2, b_1) = \frac{1}{16\pi^2}(-5, -1, 4, 233/24) \tag{39}$$

where we have normalized the $U(1)_X$-charge by 1/24. We are actually interested mainly in the coefficients $b_3$ and $b_4$. Firstly, we can see that the model maintains the $SU(3)_C$ asymptotic freedom. The coefficient $b_3$ is actually very robust. It is universal for all the models and would not change up any modification of the models as suggested in section-II.E.

Without SUSY, and without colored scalars, $b_3$ will only be more negative the asymptotic freedom stronger. $SU(4)_A$ asymptotic freedom looks uncomfortably strong. However, for any of the models to be realistic, we very likely have to put in extra scalar multiplets to take care of the various symmetry breakings. This almost necessarily changes all the coefficients except $b_3$. Following our discussion on quark mass generation in section-III.A, we consider taking $\Phi = (\mathbf{15}, \mathbf{1}, \mathbf{2}, -\mathbf{6})$ with the singlet scalars $\phi_1$ and $\phi_2$ as extra supermultiplets. This exactly kills the asymptotic freedom and gives $b_4 = 0$. But extra supermultiplets are then needed to cancel anomaly contributions from their fermionic partners. Hence we can see that without SUSY, $SU(4)_A$ is likely to have strong asymptotic freedom; while with SUSY and $\Phi = (\mathbf{15}, \mathbf{1}, \mathbf{2}, -\mathbf{6})$, it is likely to lose it totally. No conclusive statement could be made either way without a specific detailed model. Scalars like $\Phi = (\mathbf{15}, \mathbf{1}, \mathbf{2}, -\mathbf{6})$ also have a large effect on the $b_2$ coefficient and hence the scale where the $SU(3)_C$ and $SU(2)_L$ couplings meet. The possible perturbative limit on the $SU(4)_A$ coupling is hence very model dependent.

### B. The leptonic sector

Our approach in general yields a leptonic sector richer in content than the quark sector. Unlike the latter, the former is where the flexibilities in fixing the detailed fermion spectrum lie. Scheme *I* embedding gives basically an extra vector-like leptonic doublet ($L^{'}$) of hypercharge $-\mathbf{9}$, or constituent states of electric charges one and two. Again, for scheme *II* embedding, the extra doublet is just a vector-like version of the SM ones. The list of vector-like leptonic singlets is very model dependent. They come in charge zero, one or two for three of our explicit models listed in Table 4, while the only exception is *Model IA* which has charge 1/2 and 3/2 singlets [7]. A complete formulation of the symmetry breaking has to generate a realistic mass spectrum for the chiral and vector-like leptons too. This however would not be possible without fixing the contents of the sector.

We want to note about two interesting features. The first one is that extra vector-like states with the same quantum numbers as the SM leptonic singlets ($E \equiv e^{+}$) is very common.

They are in three of the explicit models, except *Model IA*. If there is no direct mass term for some or all of the SM charged leptons and they get mass only through mixing with the heavy vector-like states, the smallness of the charged lepton masses could have a natural explanation through a seesaw type mechanism.

The second feature of interest is the existence of right-handed neutrino states($N$) in the our SM-like chiral spectra. For example, there is one such state in both *Model IA* and *Model IIB*, and three of them in *Model IIA*. Right-handed neutrinos can of course have Majorana masses invariant under the SM symmetry. Their existence can lead to desirable small neutrino masses through the seesaw mechanism. In an early work on horizontal symmetry [23], it has been suggested that the right-handed neutrino mass scale is to be identified as the breaking scale of the extra (horizontal) symmetry. This suggestion could work more naturally in the models from our approach as we have fermion spectra that are full chiral (SM-like) yet able to yield states that are to be identified with right-handed neutrinos at the SM symmetry level. Moreover, these states come out in our models as a result, without their existence being assumed beforehand. Actually, they are bound to exist in any SUSY version of models from our approach, for their superpartners are the zero hypercharge scalars whose VEVs are needed for the symmetry breaking. And from the discussion in section-III.A, we can see that at least a few of them are likely to be needed for a realistic chiral fermion mass generation. The scale of right-handed neutrino Majorana masses is expected to be around $10^{13} GeV$, in a simple see-saw picture. Whether the scale is compatible with the constraints from the issues related to the RG-running of the gauge coupling is questionable. And as remarked above, such a high symmetry breaking scale almost neccessarily imply that the extra vector-like states would not give much interesting and accessible phenomenology.

We also note that effective Majorana mass for (left-handed) neutrinos from higher dimensional terms have been studied more recently from the horizontal symmetry framework [24]. The type of mechanism as an alternative for neutrino mass generation may also be relevant for models from our approach.

The right-handed neutrino states from our approach have a very interesting characteristic

feature: they can decay into a charged lepton and a meson, for example $N \longrightarrow e^+K^-$, through one of the extra charged gauge bosons as shown in Fig.3. As mentioned earlier in relation to the gauge bosons involved, at least one of the quarks in the charged meson resulted would be predominately a third family guy (see Fig.2). But any particular mass eigenstate could involved through the mass mixing. In most of the cases, the charged lepton is identifiable with the chiral leptons also only through mass mixing. Other similar decay modes might be possible through Yukawa vertices instead of gauge ones.

### C. Incoporating SUSY?

Supersymmetrizing the SM is a popular way to stablize the EW-scale. Our approach to the family structure, with the extra symmetries broken through Higgs mechanism and elementary EW-Higgs doublets, is in need of SUSY, or an alternative mechanism, to stablize the gauge hierarchy. Though the one-family SM has the very elegant chiral fermion spectrum, the Higgses needed for the symmetry breaking have to be additionally postulated. In the supersymmetrized theory, however, scalars and fermions are in general on pretty equal footing; they are partners in chiral supermultiplets. The fermionic partner of each Higgs, for example, contributes to the gauge anomalies. Our SM-like chiral fermion spectra are "derivable" basically from gauge anomaly constraints in a way similar to the one-family SM. A supersymmetrized version of any of such models would be really self-contained if the needed symmetry-breaking or mass-generating scalars (Higgses) are available in the chiral spectrum of then supermultiplets. A realistic theory of the type has to account for all the scalar masses, as well as fermion masses, may be with soft SUSY-breaking terms included. This is a very ambitious goal that we are far from achieving here; nor are we sure that it can be done within our approach. We just want to highlight some of the potentials of our models in the perspective.

In section-II.D, we have already remarked that the $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1}, \mathbf{9})$ in *Model IA* and *Model IIB*, promoted to a supermultiplet, naturally incorporates the scalar ($\phi_0$ in section-III.A)

that "extracts" $U(1)_Y$ out of the extra symmetries and gives masses to the extra quarks ($Q'$ or $Q''$). This part falls in line with the goal easily. Then in *Model IIA* and *Model IIB*, the $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}, \mathbf{3})$, together with the $(\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{6})$, could be identified as containing both the SM leptonic doublets and the Higgs/Higgsino doublets. If that works,, *Model IIB* could be really self-contained, as it is actually constructed with that as the motivation. Now, how realistic could this be?

First thing we have to note is that when both the SM lepton/slepton doublets and the Higgs/Higgsino doublets are identified from the $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}, \mathbf{3})$ and the $(\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{6})$, we do not need to have the big split in mass among the $\bar{\mathbf{4}}$ components as discussed in section-III.A. In fact, all parts of the multiplet would have to have masses at EW-scale or lower. The scalar masses can be obtained from the soft SUSY-breaking terms. But then there are also the $\mu$-terms($\mu_i L_i \bar{L}$) which lead to mixing among the states, and the fermions among the $L_i$'s have to be identified as leptons and Higgsinos afer a EW-scale matter-Higgs rotation [25]. The phenomenological implications of the type of mixing have recent analyzed by various authors recently [25,26], mainly from the perspective of MSSM. They are: violation of lepton-number in the related quark Yukawa couplings; generation of sneutrino VEV(s); mass generation for neutrino(s) through mixing with a gaugino or a Higgsino. The first one can lead to FCNC's that are potentially dangerous. However, lepton-number conserving and violating Yukawa couplings are shown to be diagonalized simultaneously, suppressing the problem. And with a alignment between the $\mu_i$ and the sneutrino VEVs, the resultant neutrino mass(es) are shown to be acceptable. To use the above mentioned result in our model and piece together a consistent picture, the details of the quark and lepton mass generations, and the scalar masses including the soft SUSY-breaking parts have to be analyzed within the framework of our extended symmetry. The scenario worths pursuing for its great esthetic appeal and is under investigation.

Given the flexibility in modifying the exact spectrum discussed in section-II.E, an alternative way to incorporate SUSY is to find a anomaly-free chiral supermutiplet spectrum that incorporates also the needed scalars, for example the $(\mathbf{15}, \mathbf{1}, \mathbf{2}, -\mathbf{6})$ and others. This

is likely to deviate from our starting theme of a SM-like chiral spectrum but might yield interesting models.

The last resort is to put in the needed scalars in vector-like pairs of the full symmetry. Or, giving up SUSY, one will have to find other workable mechanism of dynamical mass generation that is compatible with the structure of our approach and with all the experimental constraints.

## V. CONCLUDING REMARKS

We have elaborated on our new approach to the family structure of the SM proposed earlier [3]. The unique feature of our approach is to start with a SM-like chiral fermion spectrum largely "derivable" from gauge anomaly constraints within an extended symmetry of $SU(N) \otimes SU(3) \otimes SU(2) \otimes U(1)$. The most suggestive $N = 4$ case also naturally yields SM-embeddings*with the number of families being three as a result.* Model construction with $N = 3, 5$ and 6 have also been discussed. The approach has some flexibility in fixing the exact details of the spectrum, mainly in the leptonic sector. We have presented four explicit $N = 4$ models from two schemes of embedding the SM and used them to address some of the most interesting phenomenological features, concentrating on those that are more general to the approach than the more model specific ones.

The quark-sector is quite model independent, though differs between the two embedding schemes. With the use of a relative simple set of scalar multiplets, we have illustrated a possible symmetry breaking pattern that can evade the stringent FCNC constraints and naturally gives SM quark mass matrices with a hierarchical structure. The analysis flavors the scheme $I$ models a bit more. Other interesting predictions from the scenario include: a doubly-charged Higgs at EW-scale; a new vector-like quark doublet of an extra up-type and a above-the-top quark, or an extra down-type and a below-the-bottom quark, depending on the embedding scheme, possibly with relatively accessible masses; in scheme $II$ models, assuming the extra down-type quark mix with the bottom to just fix the observed $R_b$-

anomaly gives the mass of the extra quarks to be around $715 GeV$; existence of six extra charged gauge bosons; more charged leptonic singlets depending on model specifics; and likely right-handed neutrino states with interesting characteristic decay modes through the charged gauge boson channels. The RG-running of the $SU(4)_A$ coupling can go either way depending on the details of the model but asymptotic freedom for $SU(3)_C$ is always maintained.

A SUSY version of *Model IIB* could be a really self-contained model, if the Higgses are taken to be in the $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}, \mathbf{3})$ supermultiplet together with the lepton doublets. More work is needed to see if the model could then be made consistent and realistic.

Finally, we note that we have discussed in the paper a few interesting phenomenological possibilities that are not necessarily compatible with one another. Further investigation, particularly in the context of a specific model, is needed to see which ones among them can be fitted together into a single consistent model extending the SM and solving the family problem, at least theoretically.

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**Table Caption.**

Table 1: Suggestive representation structures from the standard model to $SU(N)_A \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_X$ with three quark families. Note that for $N = 3$, we have a natural trivial embedding, $U(1)_X \equiv U(1)_Y$.

Table 2: Embedding the three family SM.

Table 3: *Model IA* – representation structures, anomaly cancellations and the hypercharge embedding.

Table 4: Explicit contents of four $SU(4)_A \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_X$ models.

Table 5: Minimal $SU(5)_A \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_X$ models.

Table 6: Minimal $SU(6)_A \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_X$ models.

Table 7: Minimal $SU(3)_H \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ models.

Table 8a: A possible pattern of mass generation, starting with $\Phi = (\mathbf{15}, \mathbf{1}, \mathbf{2}, \mathbf{-6})$.

Table 8b: An alternative possible pattern of mass generation, starting with $\Phi = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}, \mathbf{3})$.

**Figure Caption.**

Figure 1: Illustrations of Froggatt-Nielsen mechanism for quark mass generation. Scheme $I$ quarks are used. Only $SU(4)_A$ and $U(1)_X$ quantum numbers shown. Two possible tree graphs that can generate the $M^{(4)}_{13}, M^{(4)}_{23}, M^{(4)}_{14}$ and $M^{(4)}_{24}$ at the second level (see Table 8a) illustrated.

Figure 2: Charged gauge boson vertices with quark singlets. Note that the quark line switch identity from a $\bar{u}_3$ to a one of the three $\bar{d}$ for scheme $I$ models; while for scheme $II$

models from $\bar{d}_3$ to a $\bar{u}$. Similar vertices involving the quark doublets always involves the $Q'$ or $Q''$.

Figure 3: Characteristic decay mode of a right-handed neutrino. Note that we have $i = 3$ or $j = 3$ for scheme $I$ or $II$ models respectively, assuming $\Phi = (\mathbf{15}, \mathbf{1}, \mathbf{2}, -\mathbf{6})$.

Table 1. Suggestive representation structures from the standard model to $SU(N)_A \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_X$ with three quark families.

| SM | $N=3$ | $N=\mathbf{4}$ | $N=5$ | $N=6$ | $N=7$ |
|---|---|---|---|---|---|
| | $(\mathbf{3},\mathbf{3},\mathbf{2})$ | $(\mathbf{4},\mathbf{3},\mathbf{2})$ | $(\mathbf{5},\mathbf{3},\mathbf{2})$ | $(\mathbf{6},\mathbf{3},\mathbf{2})$ | $(\mathbf{7},\mathbf{3},\mathbf{2})$ |
| | $(\bar{\mathbf{3}},\bar{\mathbf{3}},\mathbf{1})$ | $(\bar{\mathbf{4}},\bar{\mathbf{3}},\mathbf{1})$ | $(\bar{\mathbf{5}},\bar{\mathbf{3}},\mathbf{1})$ | $(\bar{\mathbf{6}},\bar{\mathbf{3}},\mathbf{1})$ | $(\bar{\mathbf{7}},\bar{\mathbf{3}},\mathbf{1})$ |
| | $(\bar{\mathbf{3}},\mathbf{1},\mathbf{2})$ | $(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$ | $(\bar{\mathbf{5}},\mathbf{1},\mathbf{2})$ | $(\bar{\mathbf{6}},\mathbf{1},\mathbf{2})$ | $(\bar{\mathbf{7}},\mathbf{1},\mathbf{2})$ |
| | $(\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$ | $(\bar{\mathbf{4}},\mathbf{1},\mathbf{1})$ | $(\bar{\mathbf{5}},\mathbf{1},\mathbf{1})$ | $(\bar{\mathbf{6}},\mathbf{1},\mathbf{1})$ | $(\bar{\mathbf{7}},\mathbf{1},\mathbf{1})$ |
| $(\mathbf{3},\mathbf{2})$ | | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{2})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{2})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{2})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{2})$ |
| | | | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{2})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{2})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{2})$ |
| | | | | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{2})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{2})$ |
| $(\bar{\mathbf{3}},\mathbf{1})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1})$ | | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{2})$ |
| $(\bar{\mathbf{3}},\mathbf{1})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1})$ | | | |
| | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1})$ | | | | $(\mathbf{1},\mathbf{3},\mathbf{1})$ |
| $(\mathbf{1},\mathbf{2})$ | | $(\mathbf{1},\mathbf{1},\mathbf{2})$ | | $(\mathbf{1},\mathbf{1},\mathbf{2})$ | |
| $(\mathbf{1},\mathbf{1})$ | $(\mathbf{1},\mathbf{1},\mathbf{1})$ | $(\mathbf{1},\mathbf{1},\mathbf{1})$ | $(\mathbf{1},\mathbf{1},\mathbf{1})$ | $(\mathbf{1},\mathbf{1},\mathbf{1})$ | $(\mathbf{1},\mathbf{1},\mathbf{1})$ |
| $\#Q = 1(\times 3)$ | $\#Q = 3$ | $\#Q = 4-1$ | $\#Q = 5-2$ | $\#Q = 6-3$ | $\#Q = 7-4$ |
| $\#\bar{q} = 2(\times 3)$ | $\#\bar{q} = 3+3$ | $\#\bar{q} = 4+2$ | $\#\bar{q} = 5+1$ | $\#\bar{q} = 6$ | $\#\bar{q} = 7-1$ |
| $\#L = 1(\times 3)$ | $\#L = 3$ | $\#L = 4 \pm 1$ | $\#L = 5$ | $\#L = 6 \pm 1$ | $\#L = 7$ |

Table 2. Embedding the three family SM.

| $SU(4)_A \otimes SU(3)_C \otimes SU(2)_L$ Rep. | $U(1)_X$ | $U(1)_Y$ states | 3-family $U(1)_Y$ states |
|---|---|---|---|
| $(\mathbf{4}, \mathbf{3}, \mathbf{2})$ | 1 | $\alpha - \beta \pm \gamma \quad \alpha + \beta \pm \delta$ | $(3)\ \alpha - \beta \quad \alpha + 3\beta$ |
| $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{2})$ | $a$ | $a\alpha$ | $-\alpha - 3\beta$ |
| $(\bar{\mathbf{4}}, \bar{\mathbf{3}}, \mathbf{1})$ | $x$ | $x\alpha + \beta \pm \gamma \ x\alpha - \beta \pm \delta$ | $(3)\ x\alpha + \beta \quad x\alpha - 3\beta$ |
| $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})$ | $b$ | $b\alpha$ | $x\alpha - 3\beta$ |
| $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})$ | $c$ | $c\alpha$ | $x\alpha - 3\beta$ |
| $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ | $y$ | $y\alpha + \beta \pm \gamma \ y\alpha - \beta \pm \delta$ | $(3)\ y\alpha + \beta \quad y\alpha - 3\beta$ |
| $(\mathbf{1}, \mathbf{1}, \mathbf{2})$ | $k$ | $k\alpha$ | $-y\alpha + 3\beta$ |
| $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1})$ | $z$ | $z\alpha + \beta \pm \gamma \ z\alpha - \beta \pm \delta$ | $(3)\ z\alpha + \beta \quad z\alpha - 3\beta$ |
| $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | $s$ | $s\alpha$ | $-z\alpha + 3\beta$ |

Table 3. *Model IA* – representation structures, anomaly cancellations and the hypercharge embedding.

| $SU(4)_A \otimes SU(3)_C \otimes SU(2)_L$ Rep. | $U(1)_X$ | Gauge anomalies | | | | | $U(1)_Y$ states |
|---|---|---|---|---|---|---|---|
| | | $U(1)$-grav. | $[SU(4)]^2U(1)$ | $[SU(3)]^2U(1)$ | $[SU(2)]^2U(1)$ | $[U(1)]^3$ | |
| $(\mathbf{4}, \mathbf{3}, \mathbf{2})$ | **5** | 120 | 30 | 40 | 60 | 3000 | 3 $\mathbf{1}(Q)$ $\mathbf{7}(Q')$ |
| $(\bar{\mathbf{4}}, \bar{\mathbf{3}}, \mathbf{1})$ | **1** | 12 | 3 | 4 | | 12 | 3 $\mathbf{2}(\bar{d})$ $\mathbf{-4}(\bar{u})$ |
| $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ | **-9** | -72 | -18 | | -36 | -5832 | 3 $\mathbf{-3}(L)$ $\mathbf{-9}(L')$ |
| $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1})$ | **9** | 36 | 9 | | | 2916 | 3 $\mathbf{6}(E)$ $\mathbf{0}(N)$ |
| $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{2})$ | **-14** | -84 | | -28 | -42 | -16464 | $\mathbf{-7}(\bar{Q}')$ |
| $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})$ | **-8** | -24 | | -8 | | -1536 | $\mathbf{-4}(\bar{u})$ |
| $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})$ | **-8** | -24 | | -8 | | -1536 | $\mathbf{-4}(\bar{u})$ |
| $(\mathbf{1}, \mathbf{1}, \mathbf{2})$ | **18** | 36 | | | 18 | 11664 | $\mathbf{9}(\bar{L}')$ |
| $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | **0** | | | | | | |
| *subtotal* | | 0 | 24 | 0 | 0 | -7776 | |
| $(\mathbf{6}, \mathbf{1}, \mathbf{1})$ | **-12** | -72 | -24 | | | -10368 | 3 $\mathbf{-3}(S)$ 3 $\mathbf{-9}(S')$ |
| 3 $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | **6** | 18 | | | | 648 | 3 $\mathbf{3}(\bar{S})$ |
| 3 $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | **18** | 54 | | | | 17496 | 3 $\mathbf{9}(\bar{S}')$ |
| *Total* | | 0 | 0 | 0 | 0 | 0 | |

Table 4. Explicit contents of four $SU(4)_A \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_X$ models.

| *Model IA* | | *Model IIA* | | *Model IIB* | | *Model Im* | |
|---|---|---|---|---|---|---|---|
| $(\mathbf{4},\mathbf{3},\mathbf{2},\mathbf{5})$ | 3 $\mathbf{1}(Q)$ $\mathbf{7}(Q^{'})$ | $(\mathbf{4},\mathbf{3},\mathbf{2},\mathbf{1})$ | 3 $\mathbf{1}(Q)$ $\mathbf{-5}(Q^{''})$ | $(\mathbf{4},\mathbf{3},\mathbf{2},\mathbf{1})$ | 3 $\mathbf{1}(Q)$ $\mathbf{-5}(Q^{''})$ | $(\mathbf{4},\mathbf{3},\mathbf{2},\mathbf{5})$ | 3 $\mathbf{1}(Q)$ $\mathbf{7}(Q^{'})$ |
| $(\bar{\mathbf{4}},\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$ | 3 $\mathbf{2}(\bar{d})$ $\mathbf{-4}(\bar{u})$ | $(\bar{\mathbf{4}},\bar{\mathbf{3}},\mathbf{1},\mathbf{5})$ | 3 $\mathbf{-4}(\bar{u})$ $\mathbf{2}(\bar{d})$ | $(\bar{\mathbf{4}},\bar{\mathbf{3}},\mathbf{1},\mathbf{5})$ | 3 $\mathbf{-4}(\bar{u})$ $\mathbf{2}(\bar{d})$ | $(\bar{\mathbf{4}},\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$ | 3 $\mathbf{2}(\bar{d})$ $\mathbf{-4}(\bar{u})$ |
| $(\bar{\mathbf{4}},\mathbf{1},\mathbf{2},-\mathbf{9})$ | 3 $\mathbf{-3}(L)$ $\mathbf{-9}(L^{'})$ | $(\bar{\mathbf{4}},\mathbf{1},\mathbf{2},\mathbf{3})$ | 3 $\mathbf{-3}(L)$ $\mathbf{3}(\bar{L})$ | $(\bar{\mathbf{4}},\mathbf{1},\mathbf{2},\mathbf{3})$ | 3 $\mathbf{-3}(L)$ $\mathbf{3}(\bar{L})$ | $(\bar{\mathbf{4}},\mathbf{1},\mathbf{2},-\mathbf{9})$ | 3 $\mathbf{-3}(L)$ $\mathbf{-9}(L^{'})$ |
| $(\bar{\mathbf{4}},\mathbf{1},\mathbf{1},\mathbf{9})$ | 3 $\mathbf{6}(E)$ $\mathbf{0}(N)$ | $(\bar{\mathbf{4}},\mathbf{1},\mathbf{1},-\mathbf{15})$ | 3 $\mathbf{6}(E)$ $\mathbf{12}(S^{''})$ | $(\bar{\mathbf{4}},\mathbf{1},\mathbf{1},\mathbf{9})$ | 3 $\mathbf{-6}(\bar{E})$ $\mathbf{0}(N)$ | $(\bar{\mathbf{4}},\mathbf{1},\mathbf{1},-\mathbf{15})$ | 3 $\mathbf{-6}(\bar{E})$ $\mathbf{-12}(\bar{S}^{''})$ |
| $(\mathbf{6},\mathbf{1},\mathbf{1},-\mathbf{12})$ | 3 $\mathbf{-3}(S)$ 3 $\mathbf{-9}(S^{'})$ | $(\mathbf{6},\mathbf{1},\mathbf{1},-\mathbf{6})$ | 3 $\mathbf{6}(E)$ 3 $\mathbf{0}(N)$ | $(\mathbf{6},\mathbf{1},\mathbf{1},-\mathbf{18})$ | 3 $\mathbf{6}(E)$ 3 $\mathbf{12}(S^{''})$ | | |
| $(\mathbf{1},\bar{\mathbf{3}},\mathbf{2},-\mathbf{14})$ | $\mathbf{-7}(\bar{Q}^{'})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{2},-\mathbf{10})$ | $\mathbf{5}(\bar{Q}^{''})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{2},-\mathbf{10})$ | $\mathbf{5}(\bar{Q}^{''})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{2},-\mathbf{14})$ | $\mathbf{-7}(\bar{Q}^{'})$ |
| $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1},-\mathbf{8})$ | $\mathbf{-4}(\bar{u})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1},-\mathbf{4})$ | $\mathbf{2}(\bar{d})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1},-\mathbf{4})$ | $\mathbf{2}(\bar{d})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1},-\mathbf{8})$ | $\mathbf{-4}(\bar{u})$ |
| $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1},-\mathbf{8})$ | $\mathbf{-4}(\bar{u})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1},-\mathbf{4})$ | $\mathbf{2}(\bar{d})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1},-\mathbf{4})$ | $\mathbf{2}(\bar{d})$ | $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1},-\mathbf{8})$ | $\mathbf{-4}(\bar{u})$ |
| $(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{18})$ | $\mathbf{9}(\bar{L}^{'})$ | $(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{6})$ | $\mathbf{-3}(L)$ | $(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{6})$ | $\mathbf{-3}(L)$ | $(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{18})$ | $\mathbf{9}(\bar{L}^{'})$ |
| 3 $(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{6})$ | 3 $\mathbf{3}(\bar{S})$ | $(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{24})$ | $\mathbf{-12}(\bar{S}^{''})$ | 3 $(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{24})$ | 3 $\mathbf{-12}(\bar{S}^{''})$ | $(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{24})$ | $\mathbf{12}(S^{''})$ |
| 3 $(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{18})$ | 3 $\mathbf{9}(\bar{S}^{'})$ | 3 $(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{12})$ | 3 $\mathbf{-6}(\bar{E})$ | 3 $(\mathbf{1},\mathbf{1},\mathbf{1},-\mathbf{12})$ | 3 $\mathbf{6}(E)$ | 6 $(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{12})$ | 6 $\mathbf{6}(E)$ |

Table 5. Minimal $SU(5)_A \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_X$ models.

| | *Scheme IA* | | *Scheme IB* | | *Scheme IIA* | | *Scheme IIB* | |
|---|---|---|---|---|---|---|---|---|
| | $U(1)_X$ | $Y$-states | $U(1)_X$ | $Y$-states | $U(1)_X$ | $Y$-states | $U(1)_X$ | $Y$-states |
| $(\mathbf{5},\mathbf{3},\mathbf{2})$ | 17 | 3 $\mathbf{1}(Q)$ 2 $\mathbf{7}(Q^{'})$ | 17 | 3 $\mathbf{1}(Q)$ 2 $\mathbf{7}(Q^{'})$ | 7 | 3 $\mathbf{1}(Q)$ 2 $\mathbf{-5}(Q^{''})$ | 7 | 3 $\mathbf{1}(Q)$ 2 $\mathbf{-5}(Q^{''})$ |
| $(\bar{\mathbf{5}},\bar{\mathbf{3}},\mathbf{1})$ | -2 | 3 $\mathbf{2}(\bar{d})$ 2 $\mathbf{-4}(\bar{u})$ | -2 | 3 $\mathbf{2}(\bar{d})$ 2 $\mathbf{-4}(\bar{u})$ | 8 | 3 $\mathbf{-4}(\bar{u})$ 2 $\mathbf{2}(\bar{d})$ | 8 | 3 $\mathbf{-4}(\bar{u})$ 2 $\mathbf{2}(\bar{d})$ |
| $(\bar{\mathbf{5}},\mathbf{1},\mathbf{2},)$ | -27 | 3 $\mathbf{-3}(L)$ 2 $\mathbf{-9}(L^{'})$ | 3 | 3 $\mathbf{3}(\bar{L})$ 2 $\mathbf{-3}(L)$ | 3 | 3 $\mathbf{-3}(L)$ 2 $\mathbf{3}(\bar{L})$ | 33 | 3 $\mathbf{-9}(L')$ 2 $\mathbf{-3}(L)$ |
| $(\bar{\mathbf{5}},\mathbf{1},\mathbf{1})$ | -42 | 3 $\mathbf{-6}(\bar{E})$ 2 $\mathbf{-12}(\bar{S}^{''})$ | -102 | 3 $\mathbf{-18}(\bar{T})$ 2 $\mathbf{-24}$ $(\bar{T}^{'})$ | -72 | 3 $\mathbf{12}(S^{''})$ 2 $\mathbf{18}(T)$ | -132 | 3 $\mathbf{24}$ $(T')$ 2 $\mathbf{30}(T^{''})$ |
| 2 $(\mathbf{1},\bar{\mathbf{3}},\mathbf{2})$ | -35 | 2 $\mathbf{-7}(\bar{Q}^{'})$ | -35 | 2 $\mathbf{-7}(\bar{Q}^{'})$ | -25 | 2 $\mathbf{5}(\bar{Q}^{''})$ | -25 | 2 $\mathbf{5}(\bar{Q}^{''})$ |
| $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1})$ | -20 | $\mathbf{-4}(\bar{u})$ | -20 | $\mathbf{-4}(\bar{u})$ | -10 | $\mathbf{2}(\bar{d})$ | -10 | $\mathbf{2}(\bar{d})$ |
| 2 $(\mathbf{1},\mathbf{1},\mathbf{2})$ | 45 | 2 $\mathbf{9}(\bar{L}^{'})$ | -15 | 2 $\mathbf{-3}(L)$ | 15 | $\mathbf{-3}(L)$ | -45 | 2 $\mathbf{9}(\bar{L}^{'})$ |
| $(\mathbf{1},\mathbf{1},\mathbf{2})$ | | | -15 | $\mathbf{-3}(L)$ | | | -45 | $\mathbf{9}(\bar{L}^{'})$ |
| $(\mathbf{1},\mathbf{1},\mathbf{2})$ | | | -15 | $\mathbf{-3}(L)$ | | | 15 | $\mathbf{-3}(L)$ |
| 3 $(\mathbf{1},\mathbf{1},\mathbf{1})$ | 30 | 3 $\mathbf{6}(E)$ | 30 | 3 $\mathbf{6}(E)$ | -30 | 3 $\mathbf{6}(E)$ | -30 | 3 $\mathbf{6}(E)$ |
| 3 $(\mathbf{1},\mathbf{1},\mathbf{1})$ | 30 | 3 $\mathbf{6}(E)$ | 90 | 3 $\mathbf{18}(T)$ | 60 | 3 $\mathbf{-12}(\bar{S}^{''})$ | 120 | 3 $\mathbf{-24}(\bar{T}^{'})$ |
| 2 $(\mathbf{1},\mathbf{1},\mathbf{1})$ | 60 | 2 $\mathbf{12}(S^{''})$ | 120 | 2 $\mathbf{24}(T^{'})$ | 90 | 2 $\mathbf{-18}(\bar{T})$ | 150 | 2 $\mathbf{-30}(\bar{T}^{''})$ |

Table 6. Minimal $SU(6)_A \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_X$ models.

| | *Scheme IA* | | *Scheme IB* | | *Scheme IIA* | | *Scheme IIB* | |
|---|---|---|---|---|---|---|---|---|
| | $U(1)_X$ | $Y$-states | $U(1)_X$ | $Y$-states | $U(1)_X$ | $Y$-states | $U(1)_X$ | $Y$-states |
| $(\mathbf{6},\mathbf{3},\mathbf{2})$ | -4 | 3 $\mathbf{1}(Q)$ 3 $\mathbf{7}(Q^{'})$ | -4 | 3 $\mathbf{1}(Q)$ 3 $\mathbf{7}(Q^{'})$ | 2 | 3 $\mathbf{1}(Q)$ 3 $\mathbf{-5}(Q^{''})$ | 2 | 3 $\mathbf{1}(Q)$ 3 $\mathbf{-5}(Q^{''})$ |
| $(\bar{\mathbf{6}},\bar{\mathbf{3}},\mathbf{1})$ | 1 | 3 $\mathbf{2}(\bar{d})$ 3 $\mathbf{-4}(\bar{u})$ | 1 | 3 $\mathbf{2}(\bar{d})$ 3 $\mathbf{-4}(\bar{u})$ | 1 | 3 $\mathbf{-4}(\bar{u})$ 3 $\mathbf{2}(\bar{d})$ | 1 | 3 $\mathbf{-4}(\bar{u})$ 3 $\mathbf{2}(\bar{d})$ |
| $(\bar{\mathbf{6}},\mathbf{1},\mathbf{2},)$ | 0 | 3 $\mathbf{3}(\bar{L})$ 3 $\mathbf{-3}(L)$ | 6 | 3 $\mathbf{-3}(L)$ 3 $\mathbf{-9}(L^{'})$ | 6 | 3 $\mathbf{-9}(L^{'})$ 3 $\mathbf{-3}(L)$ | 0 | 3 $\mathbf{-3}(L)$ 3 $\mathbf{3}(\bar{L})$ |
| $(\bar{\mathbf{6}},\mathbf{1},\mathbf{1})$ | 21 | 3 $\mathbf{-18}(\bar{T})$ 3 $\mathbf{-24}(\bar{T}^{'})$ | 9 | 3 $\mathbf{-6}(\bar{E})$ 3 $\mathbf{-12}(\bar{S}^{''})$ | -27 | 3 $\mathbf{24}(T^{'})$ 3 $\mathbf{30}(T^{''})$ | -15 | 3 $\mathbf{12}(S^{''})$ 3 $\mathbf{18}(T)$ |
| 3 $(\mathbf{1},\bar{\mathbf{3}},\mathbf{2})$ | 7 | 3 $\mathbf{-7}(\bar{Q}^{'})$ | 7 | 3 $\mathbf{-7}(\bar{Q}^{'})$ | -5 | 3 $\mathbf{5}(\bar{Q}^{''})$ | -5 | 3 $\mathbf{5}(\bar{Q}^{''})$ |
| 3 $(\mathbf{1},\mathbf{1},\mathbf{2})$ | 3 | 3 $\mathbf{3}(L)$ | -9 | 3 $\mathbf{9}(\bar{L}^{'})$ | -9 | 3 $\mathbf{9}(\bar{L}^{'})$ | 3 | 3 $\mathbf{-3}(L)$ |
| 3 $(\mathbf{1},\mathbf{1},\mathbf{1})$ | -6 | 3 $\mathbf{6}(E)$ | -6 | 3 $\mathbf{6}(E)$ | -6 | 3 $\mathbf{6}(E)$ | -6 | 3 $\mathbf{6}(E)$ |
| 3 $(\mathbf{1},\mathbf{1},\mathbf{1})$ | -18 | 3 $\mathbf{18}(T)$ | -6 | 3 $\mathbf{6}(E)$ | 24 | 3 $\mathbf{-24}(\bar{T}^{'})$ | 12 | 3 $\mathbf{-12}(\bar{S}^{''})$ |
| 3 $(\mathbf{1},\mathbf{1},\mathbf{1})$ | -24 | 3 $\mathbf{24}(T')$ | -12 | 3 $\mathbf{12}(S^{''})$ | 30 | 3 $\mathbf{-30}(\bar{T}^{''})$ | 18 | 3 $\mathbf{-18}(\bar{T})$ |

Table 7. Minimal $SU(3)_H \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ models.

| | *Scheme I* | *Scheme II* |
|---|---|---|
| | $U(1)_Y$-states | $U(1)_Y$-states |
| $(\mathbf{3},\mathbf{3},\mathbf{2})$ | 3 $\mathbf{1}(Q)$ | 3 $\mathbf{1}(Q)$ |
| $(\bar{\mathbf{3}},\bar{\mathbf{3}},\mathbf{1})$ | 3 $\mathbf{2}(\bar{d})$ | 3 $\mathbf{-4}(\bar{u})$ |
| $(\bar{\mathbf{3}},\mathbf{1},\mathbf{2},)$ | 3 $\mathbf{-3}(L)$ | 3 $\mathbf{-3}(L)$ |
| $(\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$ | 3 $\mathbf{-6}(\bar{E})$ | 3 $\mathbf{-12}(\bar{S}^{''})$ |
| 3 $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1})$ | 3 $\mathbf{-4}(\bar{u})$ | 3 $\mathbf{2}(\bar{d})$ |
| 3 $(\mathbf{1},\mathbf{1},\mathbf{1})$ | 3 $\mathbf{6}(E)$ | 3 $\mathbf{6}(E)$ |
| 3 $(\mathbf{1},\mathbf{1},\mathbf{1})$ | 3 $\mathbf{6}(E)$ | 3 $\mathbf{12}(S^{''})$ |

Table 8a. A possible pattern of mass generation, starting with $\Phi = (\mathbf{15}, \mathbf{1}, \mathbf{2}, -\mathbf{6})$.

| Extra singlet VEV(s) | Effective EW-doublet VEVs and mass terms generated | | | | | |
|---|---|---|---|---|---|---|
| | $\langle \Phi_{eff} \rangle$ | $SU(4)_A \otimes U(1)_X$-rep. | VEV(s) | Mass term(s) | VEV(s) | Mass term(s) |
| | $\langle \Phi \rangle$ | $15_{-6}$ | $\langle (\mathbf{1}, \mathbf{3_0})_{\mathbf{0},-\mathbf{6}} \rangle$ | $M_{33}^{(3)}$ , $M_{34}^{(4)}$ | $\langle (\mathbf{1}, \mathbf{3}_-)_{\mathbf{0},-\mathbf{6}} \rangle$ | $M_{33}^{(4)}$ |
| $\langle \phi_0 \rangle - - - \langle (\mathbf{1}, \mathbf{2}_+)_{-\mathbf{1},\mathbf{9}} \rangle$ | $\langle \Phi \rangle \, v_0/M_0$ | $15_{-6} \times \bar{4}_9 = \bar{4}_3$ | $\langle (\mathbf{1}, \mathbf{2}_+)_{-\mathbf{1},\mathbf{3}} \rangle$ | $M_{14}^{(4)}, M_{24}^{(4)}$ | $\langle (\mathbf{1}, \mathbf{2}_-)_{-\mathbf{1},\mathbf{3}} \rangle$ | $M_{13}^{(4)}, M_{23}^{(4)}$ |
| $\langle \phi_a \rangle - - - \langle (\mathbf{2}_\pm, \mathbf{1})_{\mathbf{1},-\mathbf{3}} \rangle$ | $\langle \Phi^\dagger \rangle \, v_0^* v_a/M_0^2$ | $4_{-3} \times \bar{4}_{-3} = 15_{-6}$ | $\langle (\mathbf{2}_\pm, \mathbf{2}_-)_{\mathbf{2},-\mathbf{6}} \rangle$ | $M_{31}^{(4)}, M_{32}^{(4)}$ | $\langle (\mathbf{2}_\pm, \mathbf{2}_+)_{\mathbf{2},-\mathbf{6}} \rangle$ | $M_{31}^{(3)}, M_{32}^{(3)}$ |

Table 8b. An alternative possible pattern of mass generation, starting with $\Phi = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}, \mathbf{3})$.

| Extra singlet VEV(s) | Effective EW-doublet VEVs and mass terms generated | | | | | |
|---|---|---|---|---|---|---|
| | $\langle \Phi_{eff} \rangle$ | $SU(4)_A \otimes U(1)_X$-rep. | VEV(s) | Mass term(s) | VEV(s) | Mass term(s) |
| | $\langle \Phi \rangle$ | $\bar{4}_3$ | $\langle (\mathbf{1}, \mathbf{2}_-)_{-\mathbf{1},\mathbf{3}} \rangle$ | $M_{13}^{(4)}, M_{14}^{(4)}$ | $\langle (\mathbf{1}, \mathbf{2}_+)_{-\mathbf{1},\mathbf{3}} \rangle$ | $M_{23}^{(4)}, M_{24}^{(4)}$ |
| $\langle \phi_0^\dagger \rangle - - - \langle ((\mathbf{1}, \mathbf{2}_+)_{-\mathbf{1},\mathbf{9}})^\dagger \rangle$ | $\langle \Phi \rangle \, v_0^*/M_0$ | $\bar{4}_3 \times 4_{-9} = 15_{-6}$ | $\langle (\mathbf{1}, \mathbf{3}_-)_{\mathbf{0},-\mathbf{6}} \rangle$ | $M_{33}^{(4)}$ | $\langle (\mathbf{1}, \mathbf{3_0})_{\mathbf{0},-\mathbf{6}} \rangle$ | $M_{33}^{(3)}, M_{34}^{(4)}$ |
| $\langle \phi_a \rangle - - - \langle (\mathbf{2}_\pm, \mathbf{1})_{\mathbf{1},-\mathbf{3}} \rangle$ | $\langle \Phi^\dagger \rangle \, v_a/M_0$ | $4_{-3} \times \bar{4}_{-3} = 15_{-6}$ | $\langle (\mathbf{2}_\pm, \mathbf{2}_+)_{\mathbf{2},-\mathbf{6}} \rangle$ | $M_{31}^{(3)}, M_{32}^{(3)}$ | $\langle (\mathbf{2}_\pm, \mathbf{2}_-)_{\mathbf{2},-\mathbf{6}} \rangle$ | $M_{31}^{(4)}, M_{32}^{(4)}$ |

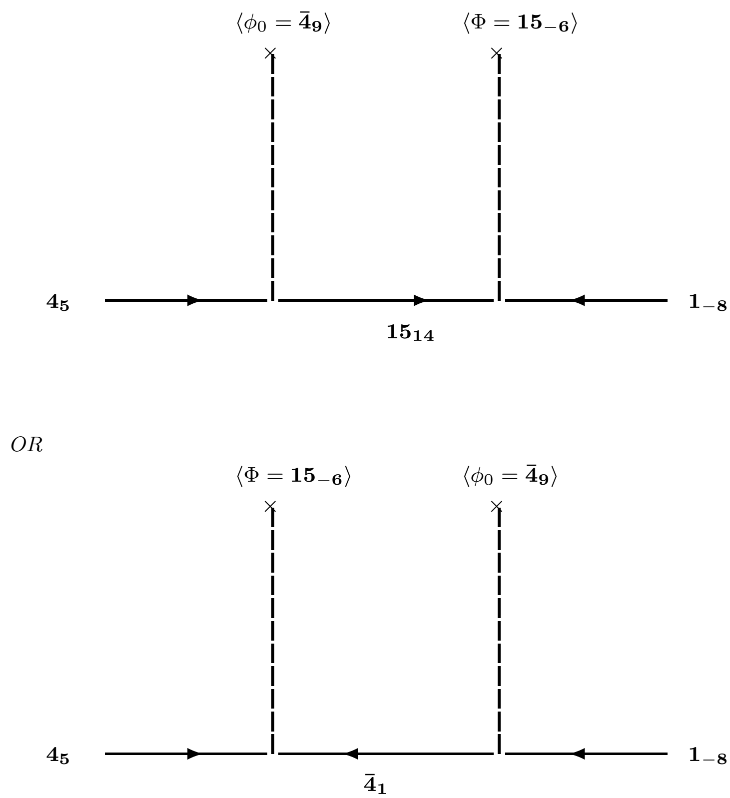


FIG. 1. Illustrations of Froggatt-Nielsen mechanism for quark mass generation.

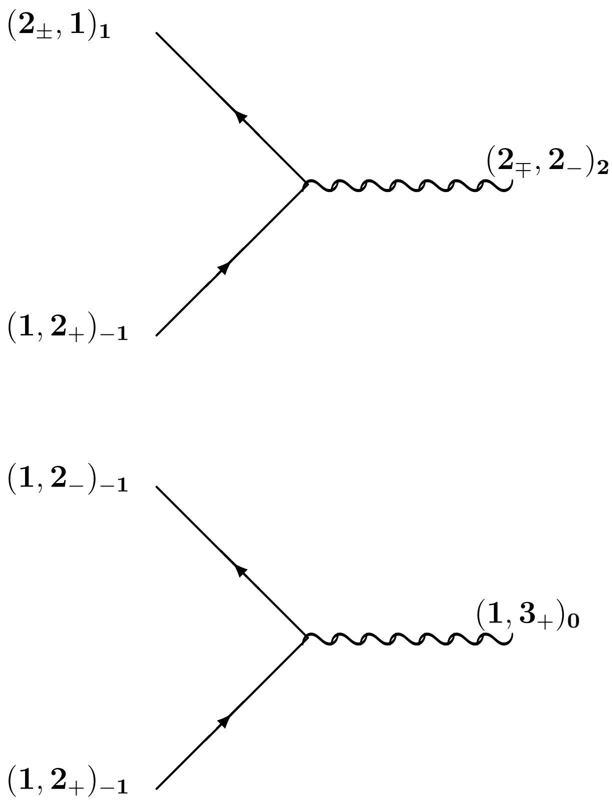


FIG. 2. Charged gauge boson vertices with quark singlets.

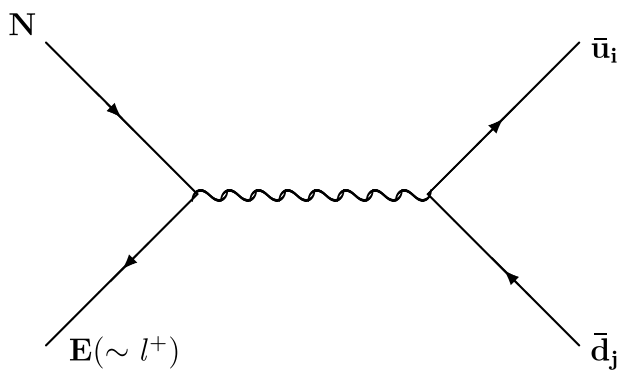


FIG. 3. Characteristic decay mode of a right-handed neutrino.